Division of Strength of Materials and Structures

Faculty of Power and Aeronautical Engineering

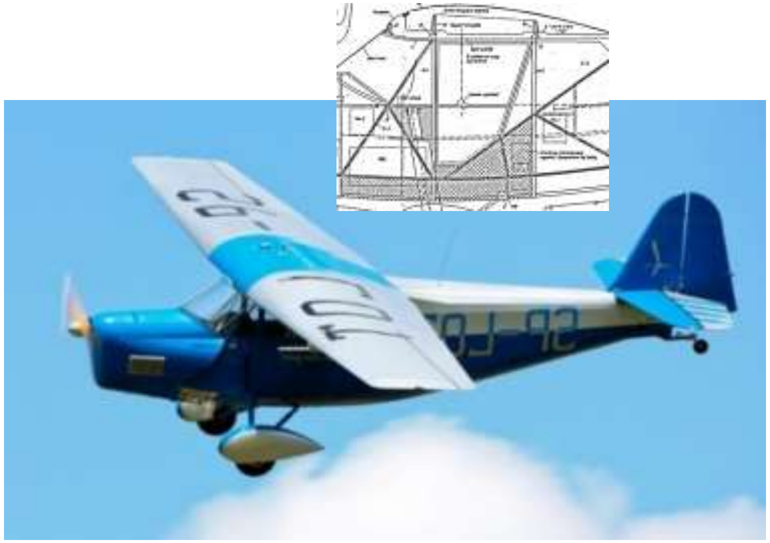
Finite element method (FEM1)

Lecture 7C. 2D_Truss element

04.2025

Examples of trusses

Bridge

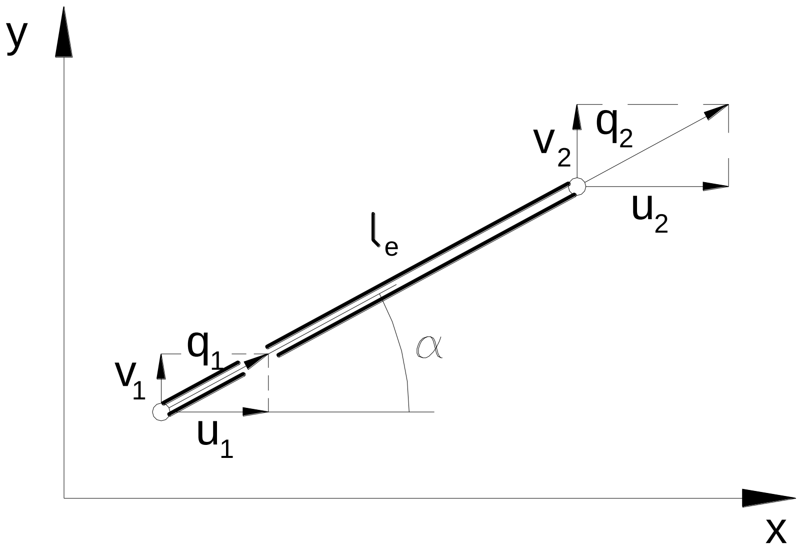


Fuselage truss

Tower crane

Roof truss

2D Truss Element



Local vector of nodal parameters: $\lfloor q \rfloor_e = \lfloor q_1, q_2 \rfloor_e$

Global vector of nodal parameters:

$$\lfloor q_g \rfloor_e = \lfloor u_1, \upsilon_1, u_2, \upsilon_2 \rfloor_e$$

Transformation of the node parameter vector:

$$q_i = u_i \cos\alpha + \upsilon_1 \sin\alpha \qquad (i = 1,2)$$

$$\boxed{\{q\}_e = [T_k]\{q_q\}_e}$$

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_e = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix} \begin{Bmatrix} u_1 \\ \upsilon_1 \\ u_2 \\ \upsilon_2 \end{Bmatrix}_e$$

Local stiffness matrix of the member:

$$\boxed{[k]_e = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

Elastic strain energy of the element:

$$U_e = \frac{1}{2}\underset{1\times 2}{\lfloor q \rfloor}\underset{2\times 2}{[k]_e}\underset{2\times 1}{\{q\}_e} = \frac{1}{2}\underset{1\times 4}{\lfloor q_q \rfloor_e^T}\underbrace{\underset{4\times 2}{[T_k]^T}\underset{2\times 2}{[k]_e}\underset{2\times 4}{[T_k]}}\underset{4\times 1}{\{q_q\}_e}$$

$$U_e = \frac{1}{2}\lfloor q_q \rfloor_e \boxed{[k_g]_e}\{q_q\}_e$$

Global truss member stiffness matrix:

$$[k_g]_e = \frac{EA}{l_e}\begin{array}{|c|c|c|c|} \hline c^2 & sc & -c^2 & -sc \\ \hline sc & s^2 & -sc & -s^2 \\ \hline -c^2 & -sc & c^2 & sc \\ \hline -sc & -s^2 & sc & s^2 \\ \hline \end{array}$$

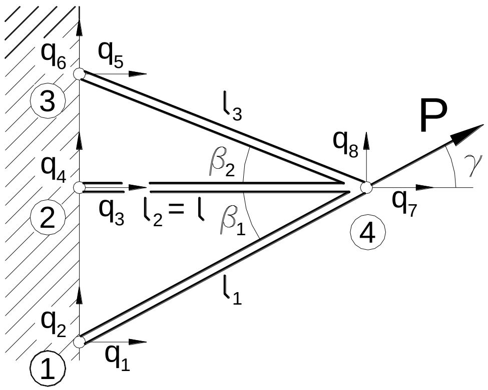
$$s = \sin\alpha,\ c = \cos\alpha$$

Example: 3-bar truss

Element 1 nodes 1 and 4 slope angle $\alpha_1 = \beta_1$ length $l_1 = \dfrac{l}{\cos\alpha_1}$.

Element 2 nodes 2 and 4 slope angle $\alpha_2 = 0$ length $l_2 = \dfrac{l}{\cos\alpha_2}$.

Element 3 nodes 3 and 4 slope angle $\alpha_3 = -\beta_2$ length $l_3 = \dfrac{l}{\cos\alpha_3}$.



$$\left[k_{ij}\right]_e^1, \left[k_{ij}\right]_e^2, \left[k_{ij}\right]_e^3$$

$$\begin{bmatrix}
k_{11}^1 & k_{12}^1 & 0 & 0 & 0 & 0 & k_{13}^1 & k_{14}^1 \\
k_{21}^1 & k_{22}^1 & 0 & 0 & 0 & 0 & k_{23}^1 & k_{24}^1 \\
0 & 0 & k_{11}^2 & k_{12}^2 & 0 & 0 & k_{13}^2 & k_{14}^2 \\
0 & 0 & k_{21}^2 & k_{22}^2 & 0 & 0 & k_{23}^2 & k_{24}^2 \\
0 & 0 & 0 & 0 & k_{11}^3 & k_{12}^3 & k_{13}^3 & k_{14}^3 \\
0 & 0 & 0 & 0 & k_{21}^3 & k_{22}^3 & k_{23}^3 & k_{24}^3 \\
k_{31}^1 & k_{32}^1 & k_{31}^2 & k_{32}^2 & k_{31}^3 & k_{32}^3 & k_{33}^1 + k_{33}^2 + k_{33}^3 & k_{34}^1 + k_{34}^2 + k_{34}^3 \\
k_{41}^1 & k_{42}^1 & k_{41}^2 & k_{42}^2 & k_{41}^3 & k_{42}^3 & k_{43}^1 + k_{43}^2 + k_{43}^3 & k_{44}^1 + k_{44}^2 + k_{44}^3
\end{bmatrix}
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ q_7 \\ q_8 \end{Bmatrix}
=
\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ P\cos\gamma \\ P\sin\gamma \end{Bmatrix}$$

Boundary conditions:

$q_j = 0 \quad j = 1,6$

$$EA\begin{bmatrix} \sum_{i=1}^{3} \dfrac{c_i^2}{l_i} & \sum_{i=1}^{3} \dfrac{s_i c_i}{l_i} \\ \sum_{i=1}^{3} \dfrac{s_i c_i}{l_i} & \sum_{i=1}^{3} \dfrac{s_i^2}{l_i} \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P\sin\gamma \\ P\cos\gamma \end{Bmatrix}$$

For: $\beta_1 = \beta_2 = \beta \quad \gamma = 0$

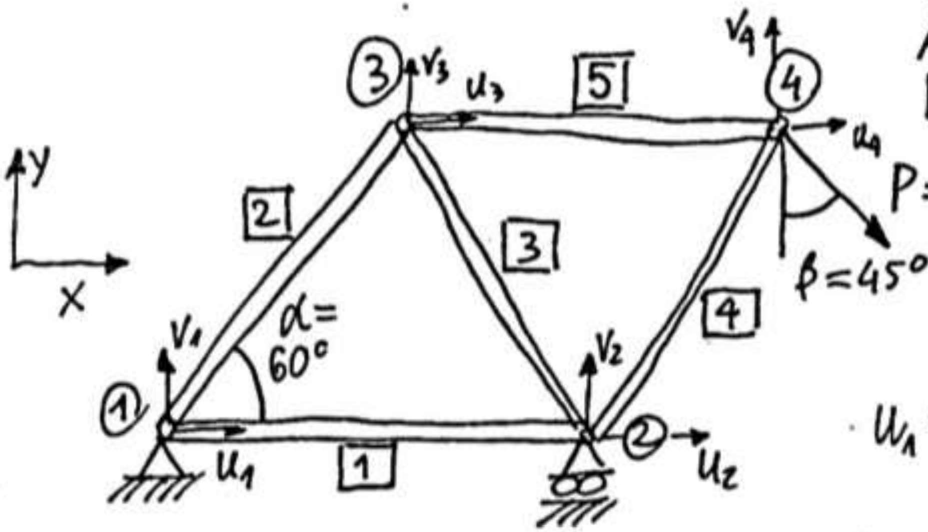
$$\frac{EA}{l}\begin{bmatrix} 1+2c^3 & 0 \\ 0 & 2s^2c \end{bmatrix} \begin{Bmatrix} q_7 \\ q_8 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

For: $c = \cos\beta$, $s = \sin\beta$

$$q_7 = \frac{Pl}{EA(1+2c^3)} \qquad q_8 = 0$$

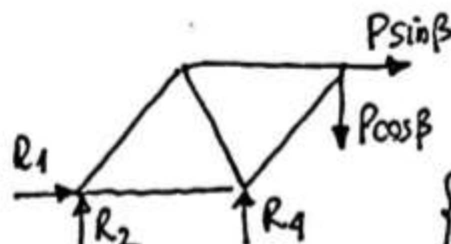
Example: Build a 2D FEM model of a truss. Find nodal displacements, stresses, internal forces and reactions.

$E = 2\cdot10^5$ MPa
$A = 100$ mm^2
$L = 1000$ mm



$u_1 = 0, \; v_1 = 0, \; v_2 = 0$

$$\underset{8\times1}{\{q\}} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix},$$

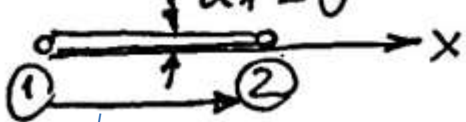


$$\underset{8\times1}{\{F\}} = \begin{Bmatrix} R_1 \\ R_2 \\ 0 \\ R_4 \\ 0 \\ 0 \\ P\sin\beta \\ -P\cos\beta \end{Bmatrix}$$

$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

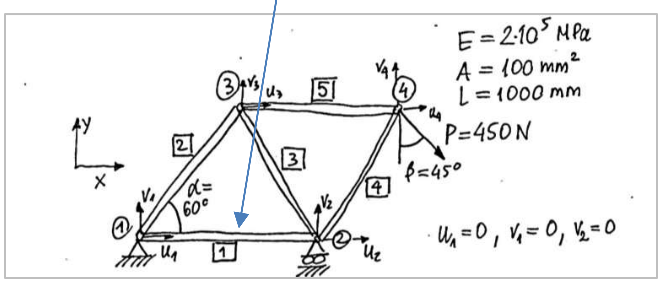
$\boxed{1}$ $\alpha_1 = 0°$



$C_1 = 1$, $S_1 = 0$, $\lfloor q_g \rfloor_1 = \lfloor u_1, v_1, u_2, v_2 \rfloor$ (1×4)

$$\underset{4\times4}{[k_g]_1} = \frac{EA}{l}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

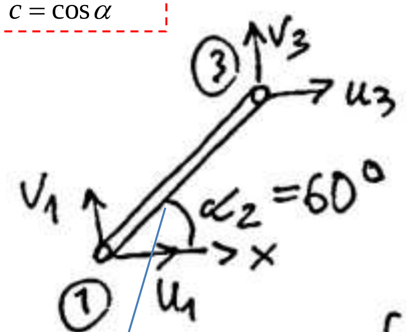
$$\underset{8\times8}{[k_g]_1^*} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

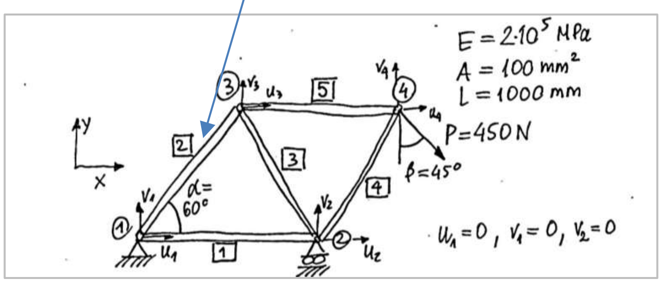
$\boxed{2}$



$$c_2 = \frac{1}{2}\ ,\quad s_2 = \frac{\sqrt{3}}{2}\ ,\quad \underset{1\times4}{\lfloor q_g \rfloor_2} = \lfloor u_1, v_1, u_3, v_3 \rfloor$$

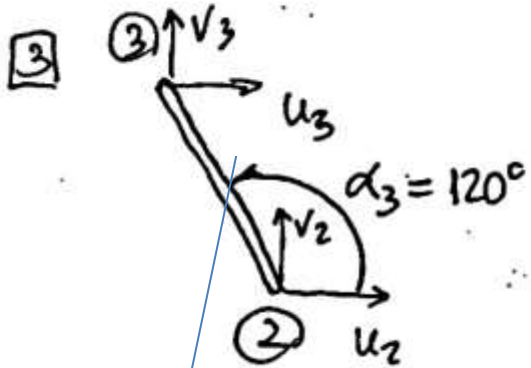
$$\underset{4\times4}{[k_g]_2} = \frac{EA}{4l}\begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times8}{[k_g]_2^*} = \frac{EA}{4l}\begin{bmatrix} 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

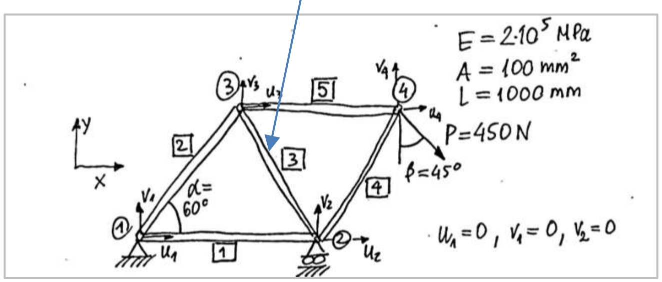
$s = \sin\alpha,\ c = \cos\alpha$



$$C_3 = -\frac{1}{2}\ , \quad S_3 = \frac{\sqrt{3}}{2}\ , \quad \lfloor q_g \rfloor_3 = \lfloor u_2, V_2, u_3, V_3 \rfloor$$

$$\underset{4\times 4}{[k_g]_3} = \frac{EA}{4L}\begin{bmatrix} 1 & -\sqrt{3} & -1 & \sqrt{3} \\ -\sqrt{3} & 3 & \sqrt{3} & -3 \\ -1 & \sqrt{3} & 1 & -\sqrt{3} \\ \sqrt{3} & -3 & -\sqrt{3} & 3 \end{bmatrix}$$

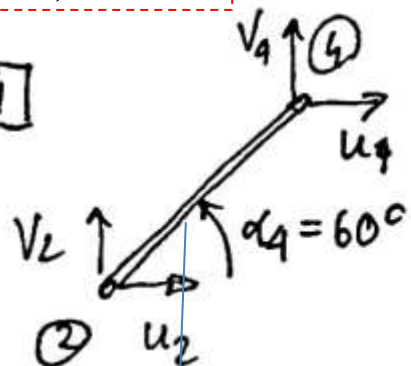
$$\underset{8\times 8}{[k_g]_3^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\sqrt{3} & -1 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & -1 & \sqrt{3} & 1 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{3} & -3 & -\sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

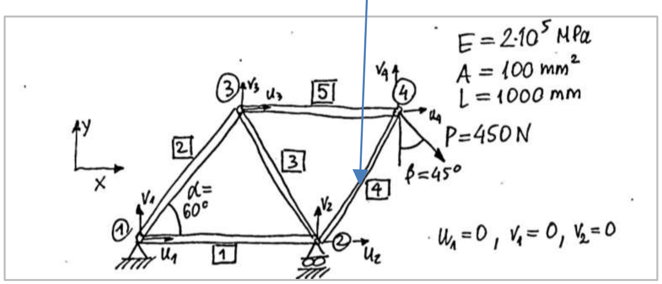
$\boxed{4}$



$$C_4 = \frac{1}{2},\quad S_4 = \frac{\sqrt{3}}{2},\quad \lfloor q_g \rfloor_4 = \lfloor u_2, v_2, u_4, v_4 \rfloor$$

$$\underset{4\times4}{[k_g]_4} = \frac{EA}{4L}\begin{bmatrix} 1 & \sqrt{3} & -1 & -\sqrt{3} \\ \sqrt{3} & 3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & 1 & \sqrt{3} \\ -\sqrt{3} & -3 & \sqrt{3} & 3 \end{bmatrix}$$

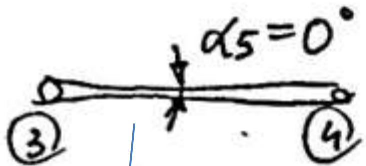
$$\underset{8\times8}{[k_g]_4^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$



$$[k_g]_e = \frac{EA}{l_e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\alpha,\ c = \cos\alpha$

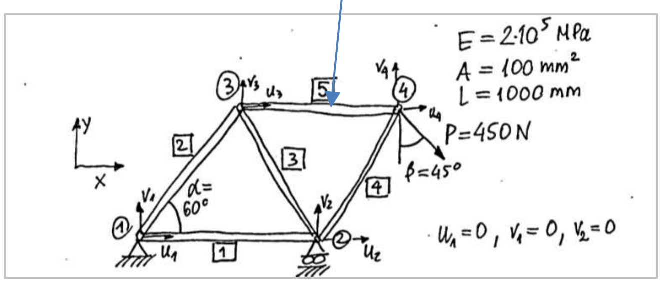
$\boxed{5}$

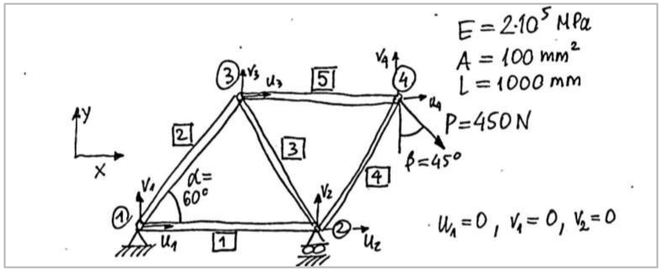


$c_5 = 1,\ s_5 = 0,\ \lfloor q_g \rfloor_5 = \lfloor u_3, v_3, u_4, v_4 \rfloor$

$$\underset{4\times 4}{[k_g]_5} = \frac{EA}{4l}\begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\underset{8\times 8}{[k_g]_5^*} = \frac{EA}{4l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$





$$\underset{8\times 8}{[k_g]_1^*} = \frac{EA}{4L}\begin{bmatrix} 4 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times 8}{[k_g]_2^*} = \frac{EA}{4L}\begin{bmatrix} 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times 8}{[k_g]_3^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\sqrt{3} & -1 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{3} & 3 & \sqrt{3} & -3 & 0 & 0 \\ 0 & 0 & -1 & \sqrt{3} & 1 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{3} & -3 & -\sqrt{3} & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

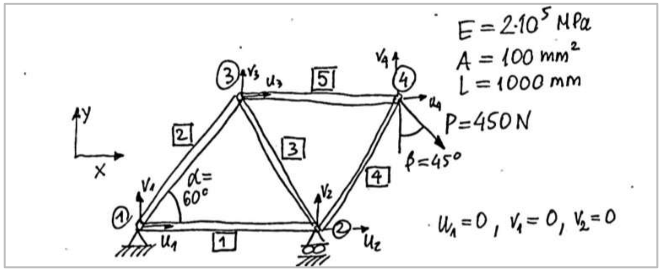
$$\underset{8\times 8}{[k_g]_4^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{3} & 0 & 0 & -1 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & 0 & 0 & 1 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times 8}{[k_g]_5^*} = \frac{EA}{4L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underset{8\times 8}{[K]} = \sum_{e=1}^{5} [k_g]_e^* = \frac{EA}{4L}\begin{bmatrix} 5 & \sqrt{3} & -4 & 0 & -1 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 3 & 0 & 0 & -\sqrt{3} & -3 & 0 & 0 \\ -4 & 0 & 6 & 0 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ 0 & 0 & 0 & 6 & \sqrt{3} & -3 & -\sqrt{3} & -3 \\ -1 & -\sqrt{3} & -1 & \sqrt{3} & 6 & 0 & -4 & 0 \\ -\sqrt{3} & -3 & \sqrt{3} & -3 & 0 & 6 & 0 & 0 \\ 0 & 0 & -1 & -\sqrt{3} & -4 & 0 & 5 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & -3 & 0 & 0 & \sqrt{3} & 3 \end{bmatrix}$$

$$\underset{8\times 8}{[K]} \cdot \underset{8\times 1}{\{q\}} = \underset{8\times 1}{\{F\}}$$

+ boundary conditions : $u_1 = 0,\ v_1 = 0,\ v_2 = 0$



$$[K]_{5\times5} \cdot \{q\}_{5\times1} = \{F\}_{5\times1}$$

$$\frac{EA}{4L}\begin{bmatrix} 6 & -1 & \sqrt{3} & -1 & -\sqrt{3} \\ -1 & 6 & 0 & -4 & 0 \\ \sqrt{3} & 0 & 6 & 0 & 0 \\ -1 & -4 & 0 & 5 & \sqrt{3} \\ -\sqrt{3} & 0 & 0 & \sqrt{3} & 3 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2}P \\ -\frac{\sqrt{2}}{2}P \end{Bmatrix}$$

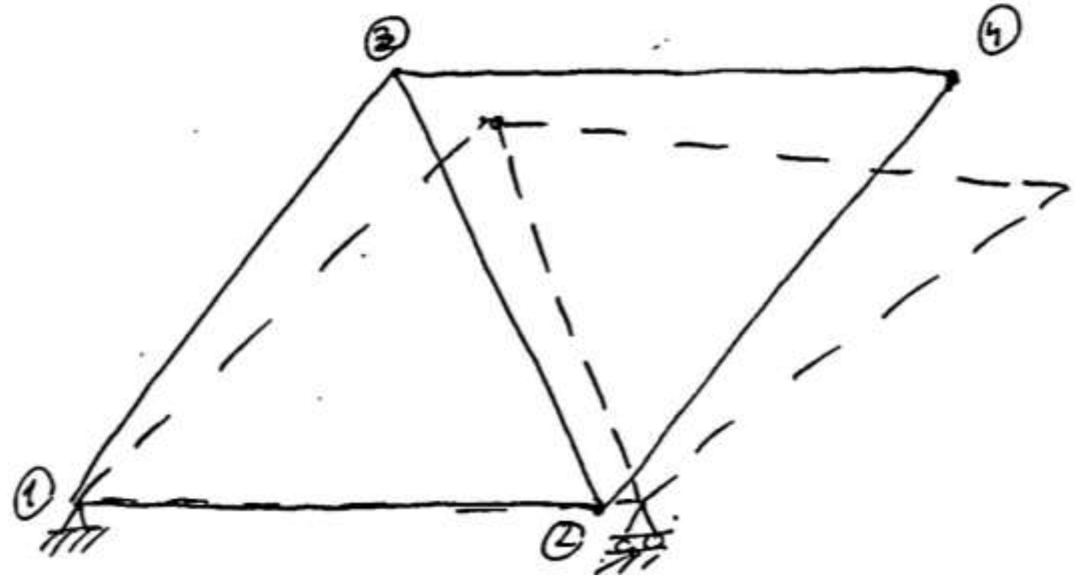
$u_2 = 0.3362 \cdot 10^{-2}$ mm

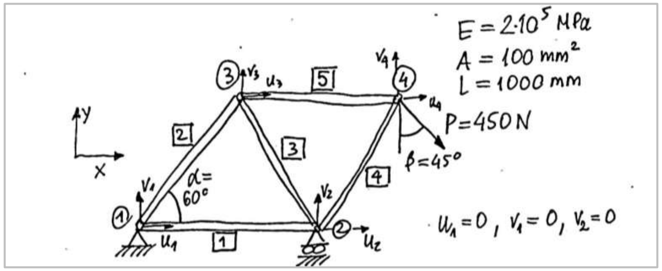
$u_3 = 5.1872 \cdot 10^{-2}$ mm

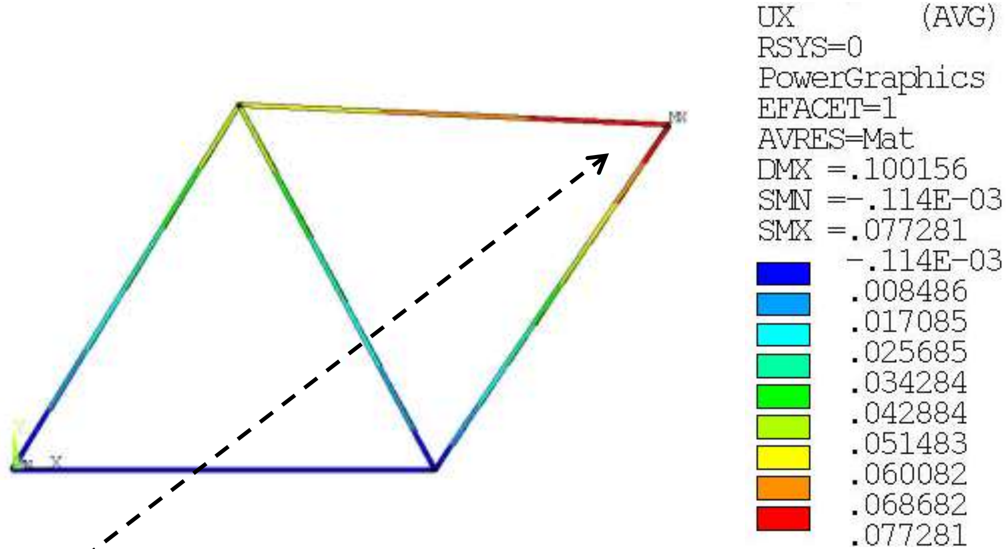
$v_3 = -0.09706 \cdot 10^{-2}$ mm

$u_4 = 7.6968 \cdot 10^{-2}$ mm

$v_4 = -6.3709 \cdot 10^{-2}$ mm







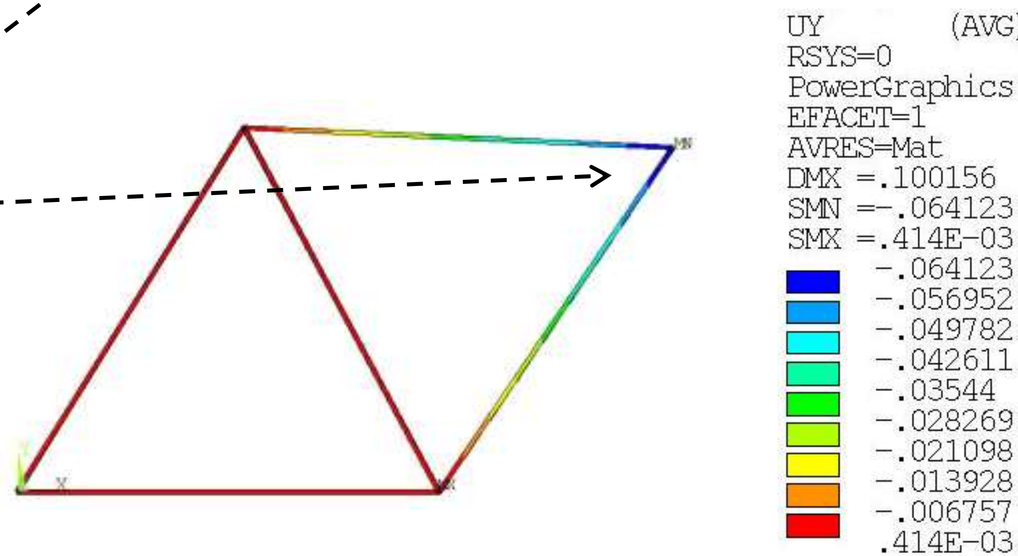
$$u_2 = 0.3362 \cdot 10^{-2}\ \text{mm}$$

$$u_3 = 5.1872 \cdot 10^{-2}\ \text{mm}$$

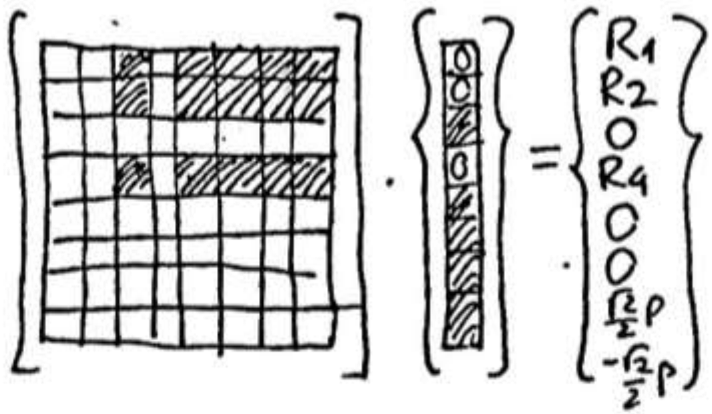
$$v_3 = -0.09706 \cdot 10^{-2}\ \text{mm}$$

$$u_4 = 7.6968 \cdot 10^{-2}\ \text{mm}$$

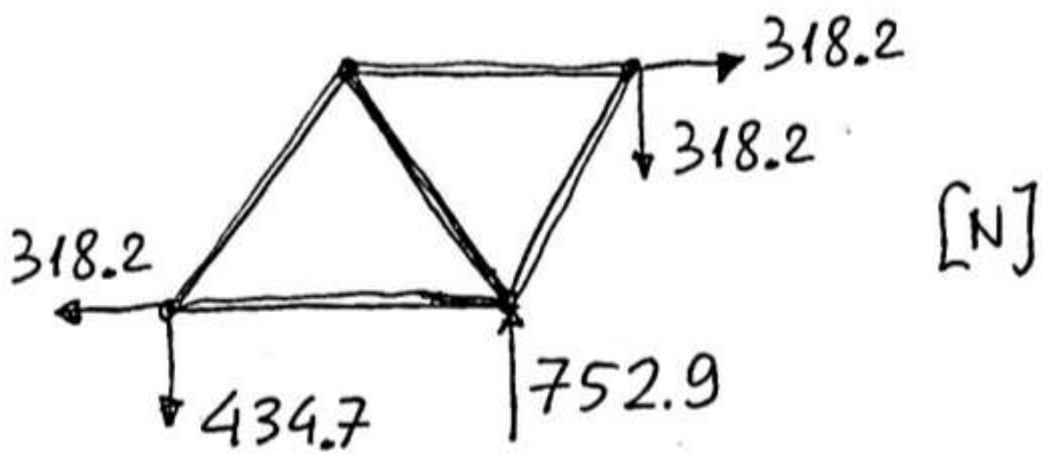
$$v_4 = -6.3709 \cdot 10^{-2}\ \text{mm}$$



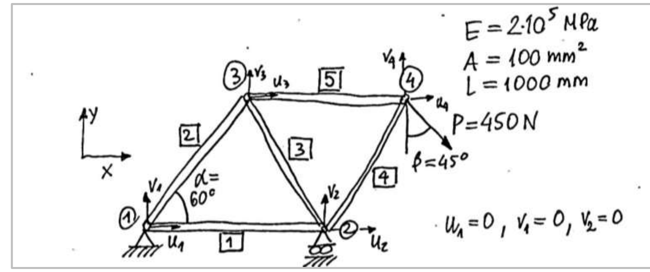
Reactions



$$\Rightarrow \quad R_1 = -318.2\,\mathrm{N}, \quad R_2 = -434.7\,\mathrm{N}, \quad R_4 = 752.9\,\mathrm{N}$$



Internal stresses and forces



$$\boxed{1}\quad \underset{2\times1}{\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_1} = \underset{2\times4}{\begin{bmatrix} T_t \end{bmatrix}_1} \cdot \underset{4\times1}{\begin{Bmatrix} q_g \end{Bmatrix}_1} = \underset{(2\times4)}{\begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & C_1 & S_1 \end{bmatrix}} \cdot \underset{(4\times1)}{\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}_1} = \begin{Bmatrix} 0 \\ 0.33622\cdot10^{-2} \end{Bmatrix}$$

$$\sigma_1 = \frac{E}{L}\left(q_2 - q_1\right)_1 = 0.67\ \text{MPa} \quad , \quad N_1 = \sigma_1 \cdot A = 67.24\ N$$

$$\boxed{2}\quad \underset{2\times1}{\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_2} = \underset{2\times4}{\begin{bmatrix} T_t \end{bmatrix}_2} \cdot \underset{4\times1}{\begin{Bmatrix} q_g \end{Bmatrix}_2} = \begin{bmatrix} C_2 & S_2 & 0 & 0 \\ 0 & 0 & C_2 & S_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix}_2 = \begin{Bmatrix} 0 \\ 2.5096\cdot10^{-2} \end{Bmatrix}$$

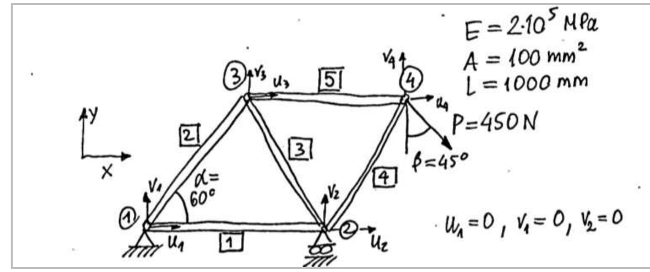
$$\sigma_2 = \frac{E}{L}\left(q_2 - q_1\right)_2 = 5.02\ \text{MPa} \quad , \quad N_2 = \sigma_2 \cdot A = 502\ N$$

$$\boxed{3}\quad \underset{2\times1}{\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_3} = \underset{2\times4}{\begin{bmatrix} T_t \end{bmatrix}_3} \cdot \underset{4\times1}{\begin{Bmatrix} q_g \end{Bmatrix}_3} = \begin{bmatrix} C_3 & S_3 & 0 & 0 \\ 0 & 0 & C_3 & S_3 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}_3 = \begin{Bmatrix} -0.16811\cdot10^{-2} \\ -2.67766\cdot10^{-2} \end{Bmatrix}$$

3 2

$$\sigma_3 = \frac{E}{L}\left(q_2 - q_1\right)_3 = -5.02\ \text{MPa} \quad , \quad N_3 = \sigma_3 \cdot A = -502\ N$$

(possible buckling)



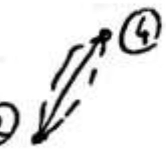
$\boxed{4}$

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_4 = [T_t]_4 \cdot \{q_g\}_4 = \begin{bmatrix} c_4 & s_4 & c & 0 \\ c & 0 & c_4 & s_4 \end{bmatrix} \cdot \begin{Bmatrix} u_2 \\ v_2 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 0.16811 \cdot 10^{-2} \\ -1.66901 \cdot 10^{-2} \end{Bmatrix}$$

2×1

$$\sigma_4 = \frac{E}{L}(q_2 - q_1)_4 = -3.67\ \text{MPa}, \quad N_4 = \sigma_4 \cdot A = -367\ \text{N}$$

(possible buckling)

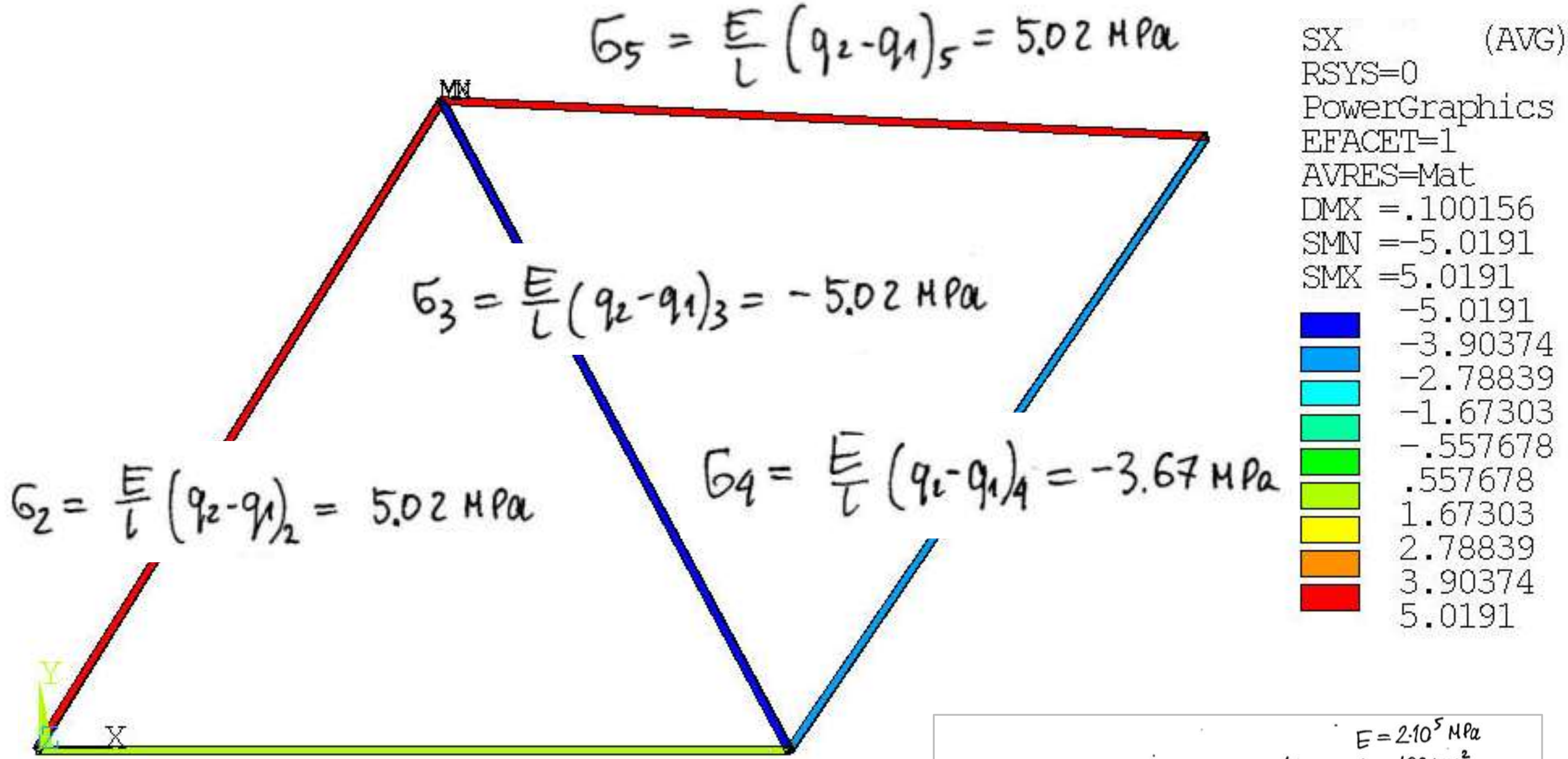


$\boxed{5}$

$$\underset{2\times1}{\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}_5} = \underset{2\times4}{[T_t]_5} \cdot \underset{4\times1}{\{q_g\}_5} = \begin{bmatrix} c_5 & s_5 & 0 & c \\ c & 0 & c_5 & s_5 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 5.18721 \cdot 10^{-2} \\ 7.6968 \cdot 10^{-2} \end{Bmatrix}$$

$$\sigma_5 = \frac{E}{L}(q_2 - q_1)_5 = 5.02\ \text{MPa}, \quad N_5 = 502\ \text{N}$$

Axial stresses [MPa]



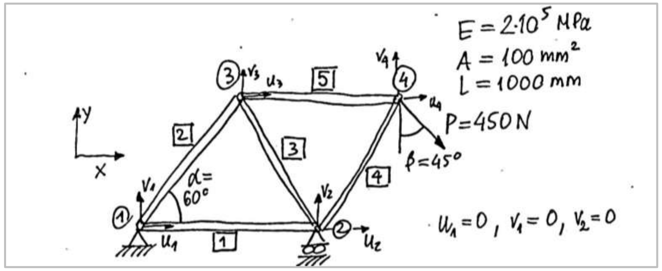
$$\sigma_5 = \frac{E}{l}(q_2 - q_1)_5 = 5.02 \text{ MPa}$$

$$\sigma_3 = \frac{E}{l}(q_2 - q_1)_3 = -5.02 \text{ MPa}$$

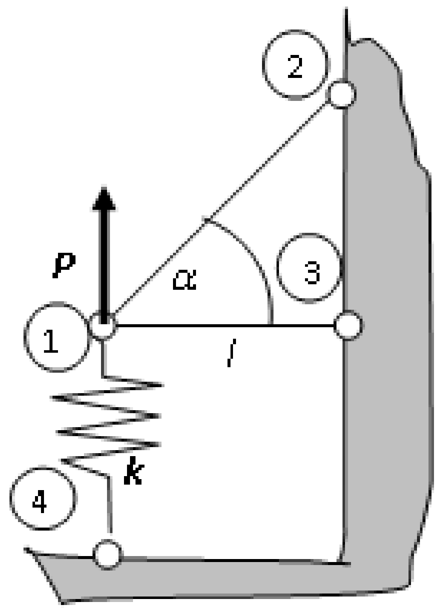
$$\sigma_4 = \frac{E}{l}(q_2 - q_1)_4 = -3.67 \text{ MPa}$$

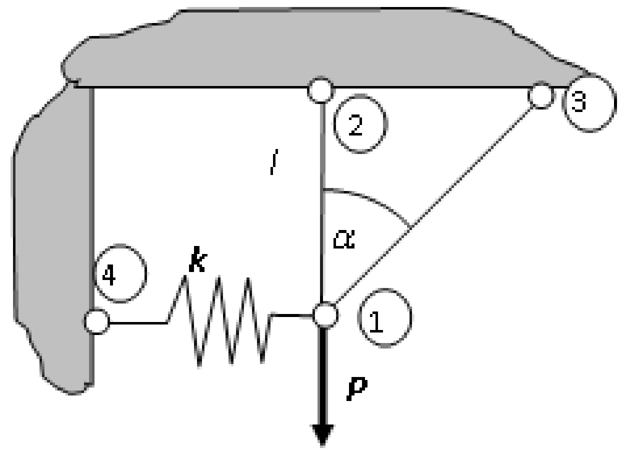
$$\sigma_2 = \frac{E}{l}(q_2 - q_1)_2 = 5.02 \text{ MPa}$$

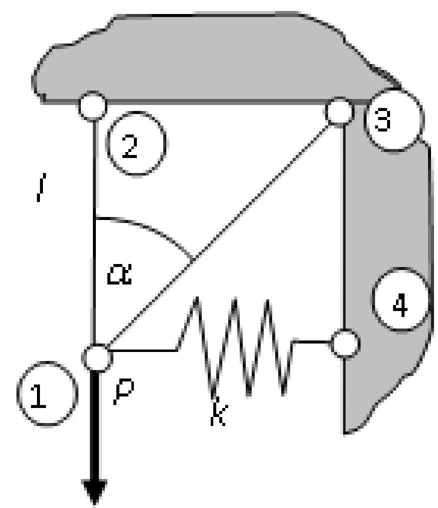
$$\sigma_1 = \frac{E}{l}(q_2 - q_1)_1 = 0.67 \text{ MPa}$$



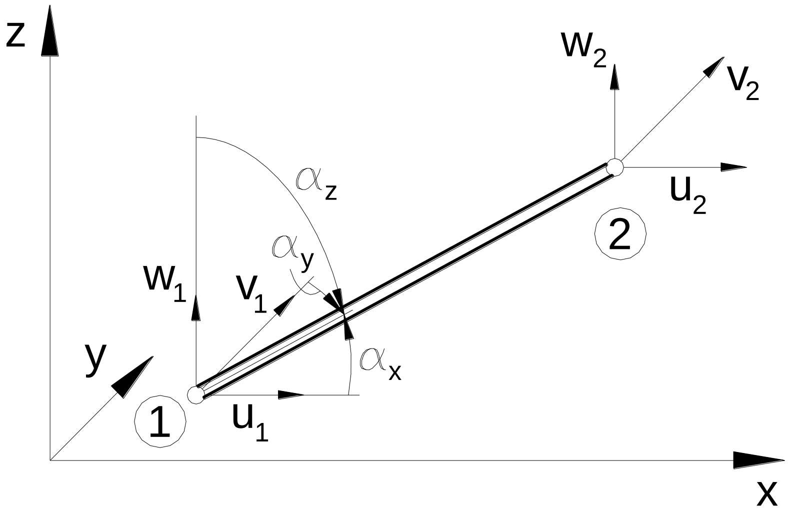
Examples of tasks







Truss member finite element in 3D



$$\{q\}_e = \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix}$$ Global vector of nodal parameters

Global truss member stiffness matrix :

$$\left[k^g\right]_e = \frac{EA}{l_e} \begin{bmatrix} c_x^2 & c_x c_y & c_x c_z & -c_x^2 & -c_x c_y & -c_x c_z \\ c_x c_y & c_y^2 & c_y c_z & -c_x c_y & -c_y^2 & -c_y c_z \\ c_x c_z & c_y c_z & c_z^2 & -c_x c_z & -c_y c_z & -c_z^2 \\ -c_x^2 & -c_x c_y & -c_x c_z & c_x^2 & c_x c_y & c_x c_z \\ -c_x c_y & -c_y^2 & -c_y c_z & c_x c_y & c_y^2 & c_y c_z \\ -c_x c_z & -c_y c_z & -c_z^2 & c_x c_z & c_y c_z & c_z^2 \end{bmatrix}$$

$c_x = \cos\alpha_x \qquad c_y = \cos\alpha_y \qquad c_z = \cos\alpha_z$